Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
| --- | --- |
| MODEL SOLUTIONS | |

Centre Number | Candidate Number

# Pearson Edexcel Level 3 GCE

## Friday 7 June 2024

Afternoon (Time: 1 hour 30 minutes) | Paper reference **9FM0/3C**

# Further Mathematics

**Advanced**

**PAPER 3C: Further Mechanics 1**

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of $g$ is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over*




P 7 5 3 2 2 A 0 1 2 4

Pearson

**1.** [*In this question,* **i** *and* **j** *are horizontal perpendicular unit vectors.*]

A particle $A$ has mass 3 kg and a particle $B$ has mass 2 kg.

The particles are moving on a smooth horizontal plane when they collide directly.

Immediately **before** the collision, the velocity of $A$ is $(3\mathbf{i} - \mathbf{j})\,\mathrm{m\,s^{-1}}$ and the velocity of $B$ is $(-6\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$

Immediately **after** the collision the velocity of $A$ is $\left(-2\mathbf{i} + \frac{2}{3}\mathbf{j}\right)\mathrm{m\,s^{-1}}$

(a) Find the total kinetic energy of the two particles **before** the collision. **(3)**

(b) Find, in terms of **i** and **j**, the impulse exerted on $A$ by $B$ in the collision. **(3)**

(c) Find, in terms of **i** and **j**, the velocity of $B$ immediately **after** the collision. **(3)**

a) Recall that $E_k = \frac{1}{2}mv^2$ and $v = |\underline{v}|$

$v_A = \sqrt{3^2 + (-1)^2} = \sqrt{10}$

$v_B = \sqrt{(-6)^2 + 2^2} = \sqrt{40}$

Total $E_k = \frac{1}{2}M_A v_A^2 + \frac{1}{2}M_B v_B^2$ (1)

$= \frac{1}{2} \times 3 \times 10 + \frac{1}{2} \times 2 \times 40$ (1)

$= 55\text{J}$ (1)

b) Recall that impulse is $\Delta(mv)$

$\Delta(mv) = M_A v_F - M_A v_I$ (1)

$= 3\begin{pmatrix} -2 \\ 2/3 \end{pmatrix} - 3\begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -15 \\ 5 \end{pmatrix}$ (1)

$= -15\underline{i} + 5\underline{j}$ Ns (1)

**Question 1 continued**

c) $I = 2V_F - 2V_I$ ①

$\Rightarrow \begin{pmatrix} 15 \\ -5 \end{pmatrix} = 2\begin{pmatrix} x \\ y \end{pmatrix} - 2\begin{pmatrix} -6 \\ 2 \end{pmatrix}$ ①

$\Rightarrow 15 = 2x + 12 \Rightarrow x = 3/2,$

$-5 = 2y - 4 \Rightarrow y = -1/2$

$\underline{V}_F = 3/2\,\underline{i} - 1/2\,\underline{j}$ ①

**(Total for Question 1 is 9 marks)**

**2.** A rough plane is inclined to the horizontal at an angle $\theta$, where $\tan\theta = \frac{3}{4}$

A particle $P$ of mass $m$ is at rest at a point on the plane.

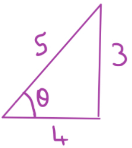
The particle is projected **up** the plane with speed $\sqrt{2ag}$

The particle moves up a line of greatest slope of the plane and comes to instantaneous rest after moving a distance $d$.

The coefficient of friction between $P$ and the plane is $\frac{1}{7}$

(a) Show that the magnitude of the frictional force acting on $P$ as it moves up the plane is $\frac{4mg}{35}$

**(3)**



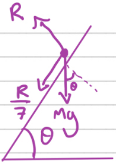

Air resistance is assumed to be negligible.

Using the work-energy principle,

(b) find $d$ in terms of $a$.

**(4)**

a)




$\tan\theta = 3/4$ $\qquad F_{max} = \mu R$

$\qquad\qquad\qquad\qquad = 1/7\, R$

$\sin\theta = 3/5$

$\cos\theta = 4/5$

Resolve perpendicular to the slope:

$R - Mg\cos\theta = 0 \Rightarrow R = Mg\cos\theta$ ①

$\Rightarrow R = \frac{4}{5}Mg$ ①

$\Rightarrow F = \frac{1}{7}R = \frac{4Mg}{35}$ ①

**Question 2 continued**

b) Work done by friction $+ E_p = E_k$

$Fd = \frac{1}{2}mv^2 + mgh$ $\qquad h = d\sin\theta$

$\frac{4mg}{35} \times d = \frac{1}{2}m \times 2ag - mg\,d\sin\theta$ ① ①

$\frac{4d}{35} = a - \frac{3}{5}d$ ①

$\Rightarrow d\left(\frac{4}{35} + \frac{3}{5}\right) = a$

$\Rightarrow d = \frac{7}{5}a$ ①

**(Total for Question 2 is 7 marks)**

3. A car of mass 1000 kg moves in a straight line along a horizontal road at a constant speed of $72\,\text{km}\,\text{h}^{-1}$

- The resistance to the motion of the car is modelled as a constant force of magnitude 900 N

The engine of the car is working at a constant rate of $P$ kW.

Using the model,

(a) find the value of $P$.

**(3)**

The car now travels in a straight line up a road which is inclined to the horizontal at an angle $\alpha$, where $\sin\alpha = \frac{2}{49}$

- In a refined model, the resistance to the motion of the car from non-gravitational forces is now modelled as a force of magnitude $20v$ newtons, where $v\,\text{m}\,\text{s}^{-1}$ is the speed of the car

At the instant when the engine of the car is working at a constant rate of 30 kW and the car is moving up the road at $10\,\text{m}\,\text{s}^{-1}$, the acceleration of the car is $a\,\text{m}\,\text{s}^{-2}$

Using the refined model,

(b) find the value of $a$.

**(4)**

Later on, when the engine of the car is again working at a constant rate of 30 kW, the car is moving up the road at a constant speed $U\,\text{m}\,\text{s}^{-1}$

Using the refined model,

(c) find the value of $U$.

**(5)**

a) Recall that $P = Fv$

$72\,\text{kmh}^{-1} = 72 \times 1000 \div 3600\,\text{ms}^{-1}$

$\Rightarrow v = 20\,\text{ms}^{-1}$ 1

Power is in W $\rightarrow 1000P = 20F$

$\Rightarrow F = 50P$

resistive force

$F - 900 = 0$ 1



P 7 5 3 2 2 A 0 6 2 4

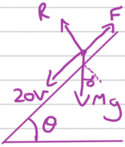
**Question 3 continued**

$\Rightarrow F = 900$

$\Rightarrow 50p = 900$

$\Rightarrow p = 18$ ①

b)




$P = Fv \Rightarrow 30{,}000 = F \times 10$

$\Rightarrow F = 3000 \text{ ms}^{-1}$

Resolve parallel to the slope ① $F = ma$

$F - 20v - Mg\sin\theta = Ma$

$3000 - 20(10) - 1000(9.8) \times \frac{2}{49} = 1000a$ ①

$\Rightarrow a = 2.4$ ①

c) $P = Fv \Rightarrow \frac{30000}{v} = F$ ①

constant speed

$\frac{30000}{v} - 20v - 1000g \times \frac{2}{49} = 0$ ①

$\Rightarrow \frac{30000}{v} - 20v - 400 = 0$ ①

$\Rightarrow 30000 - 20v^2 - 400v = 0$

$\Rightarrow v^2 + 20v - 1500 = 0$ ①

**Question 3 continued**

$\Rightarrow (v+50)(v-30)=0$

$\Rightarrow v = 30$ or $v=-50$ reject as $v \geqslant 0$

So $v = 30$ (1)

4. A particle $A$ of mass $2m$ is moving in a straight line with speed $3u$ on a smooth horizontal plane. Particle $A$ collides directly with a particle $B$ of mass $m$ which is at rest on the plane.

The coefficient of restitution between $A$ and $B$ is $e$, where $e > 0$

(a) Show that the speed of $B$ immediately after the collision is $2u(1 + e)$.

**(6)**

After the collision, $B$ hits a smooth fixed vertical wall which is perpendicular to the direction of motion of $B$.

(b) Show that there will be a second collision between $A$ and $B$.

**(3)**

The coefficient of restitution between $B$ and the wall is $\frac{1}{2}$

Find, **in simplified form**, in terms of $m$, $u$ and $e$,

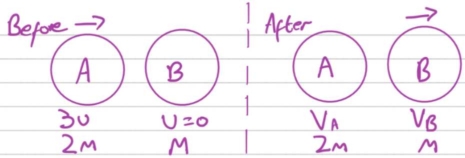
(c) the magnitude of the impulse received by $B$ in its collision with the wall,

**(3)**

(d) the loss in kinetic energy of $B$ due to its collision with the wall.

**(3)**

a)




$U_1M_1 + U_2M_2 = V_1M_1 + V_2M_2$ (COM)

By the conservation of momentum, 1

$3u \times 2m = -2mv_A + mv_B$

$\Rightarrow 6u = -2v_A + v_B$ 1 (→ positive) (assuming A goes left)

$v_2 - v_1 = e(u_2 - u_1)$ (impact law)

$v_B - (-v_A) = 3ue$ 1

$\Rightarrow 3eu = v_B + v_A$ 1





P 7 5 3 2 2 A 0 1 0 2 4

**Question 4 continued**

$\Rightarrow v_A = 3eu - v_B$ ①

$\Rightarrow 6u = -2(3eu - v_B) + v_B$

$\Rightarrow 6u = -6eu + 3v_B$

$\Rightarrow 6u(1+e) = 3v_B$

$\Rightarrow 2u(1+e) = v_B$ ①

b) $v_A = 3eu - 2u(1+e)$ ①

$\Rightarrow v_A = u(e-2)$ ①

$0 \leq e \leq 1 \Rightarrow v_A < 0 \Rightarrow$ A continues to move towards the wall. ①

c) The speed of B after the collision with the wall is $-ev_B$ (negative as opposite direction.)

$v_B' = ev_B = -u(1+e)$ ①

$I = \Delta(mv) = m(v_B' - v_B)$

$I = m(-u(1+e) - 2u(1+e))$ ①

$\Rightarrow I = 3(1+e)mu$ ①

d) $E_k = \frac{1}{2}mv^2$

$\frac{1}{2}m(v_B'^2 - v_B^2)$

$= \frac{1}{2}m((-u(1+e))^2 - (u(1+e))^2)$ ①



P 7 5 3 2 2 A 0 1 1 2 4

**Question 4 continued**

$$= \frac{1}{2} m \left(4v^2(1+e)^2 - v^2(1+e)^2\right) \quad ①$$

$$= \frac{3mv^2(1+e)^2}{2} \quad ①$$

**5.** A light elastic string has natural length $2a$ and modulus of elasticity $2mg$. One end of the string is attached to a fixed point $A$ on a horizontal ceiling. The other end is attached to a particle $P$ of mass $m$.

initial speed

The particle $P$ hangs in equilibrium at the point $E$, where $AE = 3a$.

The particle $P$ is then projected vertically downwards from $E$ with speed $\frac{3}{2}\sqrt{ag}$

Air resistance is assumed to be negligible. initially more $E_p$

no final $E_k$

Find the elastic energy stored in the string, when $P$ first comes to instantaneous rest. Give your answer in the form $kmga$, where $k$ is a constant to be found.

**(7)**


We have to consider three types of energy.

- $E_k = \frac{1}{2}mv^2$
- $E_p = mgh$
- $E_E = \frac{\lambda e^2}{2l}$

$$\frac{1}{2}mv^2 + mgh + \frac{\lambda e^2}{2l} = \frac{1}{2}mv^2 + mgh + \frac{\lambda e^2}{2l} \quad ①$$

$$\frac{1}{2}m\left(\frac{3}{2}\sqrt{ag}\right)^2 + mgx + \frac{2mga^2}{2(2a)} = 0 + 0 + \frac{2mg(x+a)^2}{2(2a)} \quad ①$$

$$\Rightarrow \frac{9}{8}mag + mgx + \frac{1}{2}mga = \frac{2mgx^2 + 4mgax + 2mga^2}{4a}$$

$$\Rightarrow \frac{9}{8}a + x + \frac{1}{2}a = \frac{x^2}{2a} + x + \frac{a}{2}$$

$$\Rightarrow \frac{9}{8}a = \frac{x^2}{2a}$$

$$\Rightarrow \frac{3a^2}{4} = x^2$$





P 7 5 3 2 2 A 0 1 4 2 4

**Question 5 continued**

$\Rightarrow x = \frac{3a}{2}$ (1) (reject $-\frac{3a}{2}$)

$\Rightarrow E_E = \frac{2mg\left(\frac{5a}{2}\right)^2}{4a}$ (1)

$\Rightarrow E_E = \frac{1}{2} mga \times \frac{25}{4}$

$\Rightarrow E_E = \frac{25mga}{8}$ (1)

**(Total for Question 5 is 7 marks)**





P 7 5 3 2 2 A 0 1 5 2 4

**6.** [*In this question,* **i** *and* **j** *are horizontal perpendicular unit vectors.*]

A particle $P$ is moving with velocity $(4\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$ on a smooth horizontal plane. The particle collides with a smooth vertical wall and rebounds with velocity $(\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$

The coefficient of restitution between $P$ and the wall is $e$.

(a) Find the value of $e$.

**(6)**

After the collision, $P$ goes on to hit a second smooth vertical wall, which is parallel to **i**.

The coefficient of restitution between $P$ and this second wall is $\frac{1}{3}$

The angle through which the direction of motion of $P$ has been deflected by its collision with this second wall is $\alpha°$.

(b) Find the value of $\alpha$, giving your answer to the nearest whole number.

**(4)**

a) Find the impulse $= \Delta(m\underline{v})$

$\underline{I} = m\left(\begin{pmatrix}1\\3\end{pmatrix} - \begin{pmatrix}4\\-1\end{pmatrix}\right)$ 1 $= m\begin{pmatrix}-3\\4\end{pmatrix}$ 1

So the impulse is in the direction $-3i + 4j$

$\sqrt{3^2+4^2} = 5 \Rightarrow \frac{1}{5}(-3i + 4j)$ 1 is the unit impulse.

dot product impulse and speed

Approach: $\frac{1}{5}\begin{pmatrix}-3\\4\end{pmatrix} \cdot \begin{pmatrix}4\\-1\end{pmatrix} = \frac{1}{5}(-12-4) = \frac{-16}{5}$ 1

Separation: $\frac{1}{5}\begin{pmatrix}-3\\4\end{pmatrix} \cdot \begin{pmatrix}1\\3\end{pmatrix} = \frac{1}{5}(-3+12) = \frac{9}{5}$ 1

Recall that $e = \frac{|\text{Separation speed}|}{|\text{approach speed}|}$

$e = \frac{9/5}{16/5} = \frac{9}{16}$ 1

**Question 6 continued**

b) The wall is parrallel to i, so the i component is unchanged.

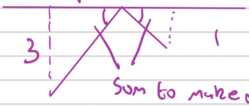
$v_j = -ev_j$ (negative as bouncing off the wall, so opposite direction)

$\underline{v}' = (1, -\frac{3}{3})$ ①

$\Rightarrow \underline{v}' = (1, -1)$ ①

$\alpha = \tan^{-1}(3) + \tan^{-1}(1)$ ①

$\Rightarrow \alpha = 117.$ ①




Sum to make $\alpha$.



P75322A01724

**7.**

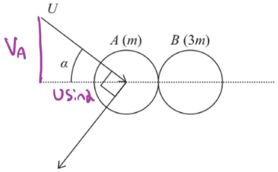



**Figure 1**

A smooth uniform sphere $A$ of mass $m$ is moving with speed $U$ on a smooth horizontal plane. The sphere $A$ collides obliquely with a smooth uniform sphere $B$ of mass $3m$ which is at rest on the plane. The two spheres have the same radius.

Immediately before the collision, the direction of motion of $A$ makes an angle $\alpha$, where $0° < \alpha < 90°$, with the line joining the centres of the spheres.

Immediately after the collision, the direction of motion of $A$ is **perpendicular** to its original direction, as shown in Figure 1.

The coefficient of restitution between the spheres is $e$.

(a) Show that the speed of $B$ immediately after the collision is

$$\frac{1}{4}(1+e)U\cos\alpha$$

**(6)**

(b) Show that $e > \frac{1}{3}$

**(4)**

(c) Show that $0 < \tan\alpha \leqslant \frac{1}{\sqrt{2}}$

**(5)**

a) Use the conservation of momentum.

$$m\begin{pmatrix} U\cos\alpha \\ U\sin\alpha \end{pmatrix} = m\begin{pmatrix} -V\sin\alpha \\ V\cos\alpha \end{pmatrix} + m\begin{pmatrix} V_B \\ 0 \end{pmatrix} \quad ①$$

$$\Rightarrow U\cos\alpha = -V\sin\alpha + V_B \quad ①$$

Use the impact law $e = \frac{V_2 - V_1}{U_1 - U_2}$

**Question 7 continued**

$(U_A - U_B)e = V_B - V_A$

$\Rightarrow Ue\cos\alpha = V_B - (-V\sin\alpha)$ 1 horizontal component.

$\Rightarrow Ue\cos\alpha = V_B + V\sin\alpha$ 1

$\Rightarrow \dfrac{Ue\cos\alpha - V_B}{\sin\alpha} = V$ 1

$\Rightarrow U\cos\alpha = -Ue\cos\alpha + 4V_B$

$\Rightarrow U\cos\alpha(1+e) = 4V_B$

$\Rightarrow \frac{1}{4}U\cos\alpha(1+e) = V_B$ 1

b) $V_a = \dfrac{Ue\cos\alpha - \frac{1}{4}U\cos\alpha(1+e)}{\sin\alpha} \sin\alpha$ 1

$= Ue\cos\alpha - \frac{1}{4}U\cos\alpha(1+e)$

$= \frac{1}{4}(3e-1)U\cos\alpha$ 1

$V_A > 0$ 1 $\Rightarrow \frac{1}{4}(3e-1)U\cos\alpha > 0$

$\Rightarrow 3e - 1 > 0$

$\Rightarrow e > \frac{1}{3}$ 1

**Question 7 continued**

c) See diagram

$$\tan\alpha = \frac{\frac{1}{4}(3e-1)U\cos\alpha}{U\sin\alpha} \quad ① \; ①$$

$$\Rightarrow \tan^2\alpha = \frac{1}{4}(3e-1) \quad ①$$

$$\Rightarrow \tan^2\alpha \leqslant \frac{1}{2} \quad ① \qquad \text{as } e \leqslant 1$$

$$\Rightarrow 0 \leqslant \tan\alpha \leqslant \frac{1}{\sqrt{2}} \quad ① \qquad \text{as } 0 \leqslant \alpha \leqslant 90$$